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Steady and Transient Analytical Solutions for Gas Flow in Porous Media with Klinkenberg Effects

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Abstract

Gas flow in porous media is different from liquid flow because of the large gas compressibility and pressure-dependent effective permeability. The latter effect, named after Klinkenberg, may have significant impact on gas flow behavior, especially in low permeability media, but it has been ignored in most of the previous studies because of the mathematical difficulty in handling the additional nonlinear term in the gas flow governing equation. This paper presents a set of new analytical solutions developed for analyzing steady state and transient gas flow through porous media with Klinkenberg effects. The analytical solutions are obtained using a new form of gas flow governing equation which incorporates the Klinkenberg effect. Additional analytical solutions for 1-D, 2-D and 3-D gas flow in porous media can be readily derived. Furthermore, the conventional assumption used for linearizing the gas flow equation has been examined for its validity and a generally-applicable procedure has been developed for accurate evaluation of the analytical solutions which use a linearized diffusivity for transient gas flow. As application examples, the new analytical solutions have been used to verify the numerical solutions, and to design new laboratory and field testing techniques to determine the Klinkenberg parameters. Also, the proposed laboratory analysis method is used to analyze laboratory data of three cores from the Geysers Coring Project, and the tests were conducted under steady-state gas flow conditions. We show that this new approach and the traditional method of Klinkenberg yield similar results of absolute permeability and Klinkenberg constant for the laboratory tests; however, the new method allows one to analyze data from both transient and steady-state tests in different flow geometries.

1. Introduction

Gas flow in porous media has recently received considerable attention because of its importance in the areas of pneumatic test analysis, contaminant remediation in the unsaturated zone, and vadose zone hydrogeology. Quantitative analysis of gas flow and its effects on contaminant transport is critical to these environmental protection and restoration projects. Therefore, analytical solutions and numerical models have been used extensively in these studies and applications.

One focus of the current research in the fields of unsaturated zone hydrology and soil physics is to develop economically feasible remediation schemes to clean up contamination in shallow aquifers. Typical contaminants in unsaturated zones are volatile organic chemicals (VOCs) and non-aqueous phase liquids (NAPLs) which have been spilled from leaking storage tanks or pipelines. Once these contaminants enter the subsurface, it is very difficult to remove them because of strong capillary and chemical forces between these contaminants and the soil particles, which is complicated by the heterogeneous nature of soils. Among currently used in-situ remediation techniques, soil vapor extraction and air sparging have proven to be very efficient and cost effective methods for the removal of VOCs or NAPLs from unsaturated soils. The successful application of these techniques depends on a thorough understanding of gas flow dynamics and site conditions. As a result, many analytical solutions (Johnson et al., 1990; McWhorter, 1990; Baehr and Hunt, 1991; Shan et al., 1992; Baehr and Joss, 1995, and Shan, 1995) and numerical models (Weeks, 1978; Wilson et al., 1987; Baehr et al., 1989; Mendoza and Frind, 1990; Pruess 1991; Falta et al., 1992; Huyakorn et al., 1994, Panday et al., 1995) have been developed for analyzing gas flow in the unsaturated zone.

The systematic investigation of gas flow in porous media was pioneered in the petroleum industry for the development of natural gas reservoirs (Muskat, 1946). It has been a standard technique in the petroleum industry to use gas flow models in natural gas production and for the estimation of formation gas permeability and other reservoir parameters (Dake, 1978; and Ikoku, 1984). There exists a considerable amount of studies on the theory and application of isothermal flow of gases through porous media in the petroleum literature. The earliest attempt to solve gas flow problems used the method of successions of steady states proposed by Muskat (1946). Approximate analytical solutions (Katz et al., 1959) were then obtained by linearizing the flow equation for an ideal gas to yield a diffusion-type equation. Such solutions were found to be of limited general use because of the assumption introduced to simplify the gas properties and the flow equation. The reasons are that, in general, gas flow in reservoirs does not follow the ideal gas law, and the variations of pressure around gas production wells are too large to use constant properties. It was not until the mid sixties that more reliable mathematical solutions were developed using a numerical method (Russell et al., 1966) and introducing a real gas pseudo-pressure function (Al-Hussainy et al., 1966).

In recent years, hydrologists and soil scientists have applied similar techniques, as used in the petroleum industry, to conduct soil characterization studies by pneumatic testing of air flow properties. Pneumatic test analysis has become an important methodology in determining formation properties of two-phase unsaturated-zone flow in a proposed repository of high-level radionuclear waste (Ahlers et al., 1995). Because the ideal gas law is a better approximation to the near surface air flow than in deep gas reservoirs and also the pressure changes in the unsaturated zone are generally small, the simple linearization using an ambient, averaged gas pressure in evaluating the gas diffusivity term in the flow equation may be suitable for unsaturated-zone applications.

While the numerical models developed can be used to perform rigorous modeling studies of gas flow under complex conditions, the analytical solutions have continued to provide a simple tool to determine gas flow properties. Despite the progress made so far in our understanding of porous medium gas flow, one important aspect, the Klinkenberg effect (Klinkenberg, 1941), has been ignored in most studies. Even though efforts have been made to estimate errors introduced by neglecting the Klinkenberg effect (Baehr and Hult, 1991), only few studies are available for addressing this phenomenon or providing analytical tools. Gas flow in porous media behaves differently from liquid flow; first because gas is highly compressible compared to a liquid, and second because of the Klinkenberg effect. The Klinkenberg effect may have significant impact on gas flow behavior, especially in low permeability media. Some recent laboratory studies (Reda, 1987; Persoff and Hulen, 1995) concluded that the Klinkenberg effect is important in the low permeability formation studied and cannot be ignored.

According to Klinkenberg (1941), effective gas permeability at a finite pressure is given by

$$k_g = k_\infty \left(1 + \frac{b}{P}\right) \tag{1.1}$$

where k_∞ is the absolute, gas-phase permeability under very large gas-phase pressure at which condition the Klinkenberg effects are negligible; and b is the Klinkenberg factor, dependent on the pore structure of the medium and temperature for a given gas. Physically, Klinkenberg effects are significant in any situation where the mean free path of gas molecules in porous media approaches the pore dimension, i.e., when significant molecular collisions are with the pore wall rather than with other gas molecules. Gas permeability is then enhanced by "slip flow". Therefore, it has been expected that the Klinkenberg effect is greatest in fine-grained, lower permeable porous media. Jones (1972) found that b generally decreases with increasing permeability according to

$$b \propto k_{\infty}^{-0.36} \tag{1.2}$$

based on a study using 100 cores ranging in permeability from 0.01 to 1000 md. Typical values of b may be estimated as listed in Table 1.

Table 1. Typical values of the Klinkenberg factor, b.

k (m^2)	b (Pa)
10^{-12}	3.95×10^{3}
10^{-15}	4.75×10^{4}
10^{-18}	7.60×10^{5}

This paper presents a set of analytical solutions developed to analyze steady-state and transient gas flow through porous media with Klinkenberg effects. We have derived a new variable (pressure function) to simplify the gas flow governing differential equation with the Klinkenberg effect. In term of the new variable, the gas flow equation has the same form as that without including the Klinkenberg effect under the same linearization assumption. As a result, many 1-D, 2-D and 3-D gas flow solutions can be readily derived by analogy to single-phase slightly compressible liquid flow or heat conduction problems.

As examples of application, the analytical solutions have been used to verify the numerical solutions for simulating Klinkenberg effects and to provide linear correlations according to which data can be plotted to determine the values of k_{∞} and b. For steady-state flow in a linear system, another set of values for k_{∞} and b can also be determined by plotting k_g, the effective gas permeability against the inverse of the mean of the inlet and outlet pressures. This is the traditional method of analyzing Klinkenberg-affected gas flow (Scheidegger, 1974), and it can be used to examine the new method.

To demonstrate the application of the proposed laboratory technique to determining the Klinkenberg parameters, steady-state, single-phase gas flow tests have been conducted using three core plugs of Graywacke from well NEGU-17 of The Geysers geothermal field in California. The gas permeability measurements are analyzed using the proposed method, and consistent results have been obtained for Klinkenberg coefficients.

2. Gas Flow Equation with Klinkenberg Effects

Under isothermal conditions, gas flow in porous media is governed by a mass balance equation,

$$\nabla \bullet \left(\rho \vec{v} \right) = -f \frac{\partial(\rho)}{\partial t} \tag{2.1}$$

where ρ is gas density; ϕ is formation porosity, assumed to be constant; $\vec{v}$ is the Darcy's velocity of the gas phase, defined as

$$\vec{v} = -\frac{k_g}{\mu}(\nabla P - \rho \vec{g}) \tag{2.2}$$

where μ is gas-phase viscosity; P is gas-phase pressure, $\vec{g}$ is gravity vector; and k_g is effective gas-phase permeability, described by Equation (1.1), including the Klinkenberg effects.

The ideal gas law is here used to describe the relation between gas density and pressure as,

$$\rho = \beta P \tag{2.3}$$

where β is a compressibility factor, defined as

$$\beta = M_g / RT \tag{2.4}$$

with M_g being the molecular weight of the gas; R the universal gas constant; and T constant temperature.

When gravity effects are ignored, combining Equations (2.1) - (2.3), and (1.1) will give

$$\nabla \bullet \left(\frac{k_\infty \beta}{\mu} (P + \mathrm{b})(\nabla P) \right) = -\phi \beta \frac{\partial P}{\partial t} \tag{2.5}$$

Now we introduce a new variable, the pressure function:

$$P_b = P + b \tag{2.6}$$

In terms of the new variable, Equation (2.5) may be written as,

$$\nabla \bullet \left(\nabla P_b^2 \right) = \frac{1}{\alpha} \frac{\partial P_b^2}{\partial t} \tag{2.7}$$

where α is a gas diffusivity, defined as a function of gas pressure,

$$\alpha = \frac{k_\infty P_b}{\phi \mu} \tag{2.8}$$

It is interesting to note that Equation (2.7) is identical to the gas flow governing equation which does not include the Klinkenberg effects with P_b being replaced by P.

3.0 Analytical Solutions

The gas flow equation (2.7) is a non-linear partial differential equation with respect to P_b^2 because of the diffusivity term α, which is a function of pressure (2.8). In general, the gas flow governing equation (2.7) needs to be solved by a numerical method. However, certain analytical solutions will be possible to be obtained as proven in the following flow conditions.

3.1 Steady-State Solutions

a. Linear Flow

Under one-dimensional, linear, horizontal and steady state flow conditions, Equation (2.5) can be expressed as

$$\frac{\partial}{\partial x}\left(\frac{k_\infty \beta}{\mu}(P+b)\frac{\partial P}{\partial x}\right) = 0 \tag{3.1}$$

The boundary conditions are: at the inlet (x=0), a constant mass injection rate q_m per unit cross-sectional area is imposed,

$$-\frac{k_\infty \beta}{\mu}(P+b)\frac{\partial P}{\partial x} = q_m \qquad \text{for } x=0 \qquad (3.2)$$

and at the outlet ($x=L$), the pressure is kept constant,

$$P=P_L \qquad \text{for } x=L \qquad (3.3)$$

Then, a steady state solution of Equations (3.1), (3.2), and (3.3) can be derived for gas pressure distribution along the linear rock column as,

$$P(x) = -b + \sqrt{b^2 + P_L^2 + 2bP_L + 2q_m\mu(L-x)/k_\infty \beta} \qquad (3.4)$$

b. Radial Flow

Under one-dimensional, radial, horizontal, and steady state flow condition, Equation (2.5) becomes

$$\frac{\partial}{\partial r}\left(\frac{k_\infty \beta}{\mu} r(P+b)\frac{\partial P}{\partial r}\right) = 0 \qquad (3.5)$$

The boundary conditions are: at the well, inner boundary ($r=r_w$), a constant mass pumping (or injection) rate Q_m is imposed,

$$\frac{2\pi k_\infty h r_w \beta}{\mu}(P+b)\frac{\partial P}{\partial r} = Q_m \qquad \text{for } r=r_w \qquad (3.6)$$

where h is the thickness of the formation. At the outer boundary ($r=r_e$), the pressure is kept constant

$$P=P_e \qquad \textit{for } r=r_e \qquad (3.7)$$

Then, a steady state solution of Equations (3.5), (3.6), and (3.7) can be derived for gas pressure distribution in the radial direction,

$$P(r) = -b + \sqrt{b^2 + P_e^2 + 2bP_e + Q_m \mu \ln(r_e/r)/\pi r_w h k_\infty \beta} \qquad (3.8)$$

3.2 Transient Solutions

Equation (2.7) may be linearized using the conventional approach for transient gas flow analysis, i.e., set

$$\alpha = \frac{k_\infty P_b}{\phi\mu} \approx \frac{k_\infty \overline{P}_b}{\phi\mu} \tag{3.9}$$

where $\overline{P}_b = \overline{P} + b$, is a function of average gas pressure, treated as a constant. With the approximation of (3.9), Equation (2.7) becomes linear with respect to P_b^2, and many analytical solutions can be obtained by analogous analysis of heat conduction problems (Carslaw and Jaeger, 1959) as follows.

a. Flow in Linear Semi-Infinite Systems

In a semi-infinite system, for one-dimensional, linear, horizontal, and transient flow, Equation (2.7) becomes

$$\frac{\partial^2 P_b^2}{\partial x^2} = \frac{1}{\alpha}\frac{\partial P_b^2}{\partial t} \tag{3.10}$$

The initial condition of the system is at uniform pressure P_i,

$$P = P_i \qquad \text{for } t = 0,\ x > 0 \tag{3.11}$$

There are two types of boundary conditions at the inlet $x=0$, first a constant mass injection (pumping) rate q_m per unit area is imposed as,

$$-\frac{k_\infty \beta}{\mu}(P+b)\frac{\partial P}{\partial x} = q_m \qquad \text{for } t > 0,\ x=0 \tag{3.12}$$

and the second is a constant gas pressure,

$$P = P_0 \qquad \text{for } t > 0,\ x = 0 \tag{3.13}$$

At a distance far away from the inlet, the pressure is not disturbed and maintained at the initial value,

$$P = P_i \qquad \text{for } x \to \infty \tag{3.14}$$

In terms of the new variable of (2.6), Equations (3.11) - (3.14) can be written as follows.

The initial condition:

$$P_b^2 = (P_i + b)^2 = P_{bi}^2 \qquad \text{for } t = 0, x > 0 \quad (3.15)$$

The mass flux inlet boundary:

$$-\frac{k_\infty \beta}{2\mu}\frac{\partial P_b^2}{\partial x} = q_m \qquad \text{for } x=0, \; t > 0 \quad (3.16)$$

The constant pressure inlet boundary:

$$P_b^2 = (P_0 + b)^2 = P_{b0}^2 \qquad \text{for } t > 0, x = 0 \quad (3.17)$$

At a large distance from the inlet,

$$P_b^2 = (P_i + b)^2 = P_{bi}^2 \qquad \text{for } x \to \infty \quad (3.18)$$

Then, the solution of Equations (3.10), (3.15), (3.16), and (3.18) for the flux condition at the inlet can be found from Carslaw and Jaeger (1959, pp. 75) as,

$$P_b^2(x,t) = P_{bi}^2 + \frac{4\mu q_m}{k_\infty \beta}\left\{\sqrt{\frac{\alpha t}{\pi}}\, exp(-x^2/4\alpha t) - \frac{x}{2}\, erfc\frac{x}{2\sqrt{\alpha t}}\right\} \quad (3.19).$$

The solution of Equations (3.10), (3.15), (3.17), and (3.18) for the constant pressure condition at the inlet can be found from Carslaw and Jaeger (1959, pp.60) as,

$$P_b^2(\mathrm{x},\mathrm{t}) = P_{bi}^2 + \left(P_{b0}^2 - P_{bi}^2\right)\left\{erfc\frac{x}{2\sqrt{\alpha t}}\right\} \quad (3.20).$$

The mass flux at the inlet (x=0) can then be calculated as

$$q_{m0} = k_\infty \beta\left(P_{b0}^2 - P_{bi}^2\right)/2\mu\sqrt{\pi \alpha t} \quad (3.21)$$

b. Flow in Radial Systems

Similarly to the above 1-D linear flow case, for horizontal radial flow towards a well in an infinite, uniform, and horizontal formation, the gas flow equation, (2.7), may be expressed in terms of P_b^2,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_b^2}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial P_b^2}{\partial t} \tag{3.22}$$

with the uniform initial condition:

$$P_b^2=\left(P_i+b\right)^2=P_{bi}^2 \qquad \text{for } t=0,\ r>0 \tag{3.23}$$

The two well boundary conditions proposed are: (1) a line source/sink well:

$$\lim_{r\to 0}\frac{\pi k_\infty h r\beta}{\mu}\frac{\partial P_b^2}{\partial r}=Q_m \tag{3.24}$$

and (2) a constant wellbore pressure,

$$P_b^2=\left(P_w+b\right)^2=P_{bw}^2 \qquad \text{for } t>0,\ r=r_w \tag{3.25}$$

At the large distance from the well, the pressure is kept constant, i.e.,

$$P_b^2=\left(P_i+b\right)^2=P_{bi}^2 \qquad \text{for } r\to\infty \tag{3.26}.$$

With the linearization to the diffusivity α, (3.9), the problem of Equations (3.22), (3.23), (3.24), and (3.26) for the line source/sink case is identical to the Theis solution (Theis, 1935) in terms of P_b^2. The solution is then written as,

$$P_b^2(r,t)=P_{bi}^2-\frac{\mu q_m}{2\pi k_\infty h\beta}Ei\left(-\frac{r^2}{4\alpha t}\right) \tag{3.27}.$$

For the case of flow under the constant wellbore pressure, the problem of Equations (3.22), (3.23), (3.25), and (3.26) for the line source/sink case is identical to the heat conduction problem (Carslaw and Jaeger, 1959, pp. 335) in terms of P_b^2. The solution for pressure is,

$$P_b^2(r,t)=P_{bw}^2-\frac{2\left(P_{bw}^2-P_{bi}^2\right)}{\pi}\int_0^\infty exp\left(-\alpha u^2 t\right)\frac{J_0(ur)Y_0(ur_w)-J_0(ur_w)Y_0(ur)}{J_0^2(ur_w)+Y_0^2(ur_w)}\frac{du}{u} \tag{3.28}$$

where J_0 and Y_0 are the Bessel functions of order 0 of the first and second kind, respectively. The mass flux of gas at the wellbore can be obtained,

$$q_{mw}=\frac{k_\infty\beta\left(P_{bw}^2-P_{bi}^2\right)}{r_w\pi^2\mu}\int_0^\infty\frac{exp\left(-\alpha u^2 t\right)}{J_0^2(ur_w)+Y_0^2(ur_w)}\frac{du}{u} \tag{3.29}.$$

4. Evaluation of Analytical Solutions

The steady-state solutions derived above are exact solutions, and can be directly applied to analyzing gas flow under steady state flow conditions. However, the transient solutions of gas flow provided in Section 3 are approximate solutions because they are based on a critical assumption, i.e., using a constant diffusivity, Equation (3.9), to linearize the gas flow equation, (3.10) or (3.22). Such solutions, though widely used in the analysis of transient gas flow in unsaturated zones (Weeks, 1978; and Shan, 1995), need to be further investigated for the validity of the linearization assumption and for the conditions under which these solutions apply. In the petroleum literature, it has been found that in many situations the linearization assumption is inappropriate when applied to the flow of a real gas in gas reservoirs (Dake, 1978; Ikoku, 1984; Al-Hussainy et al., 1966; and Russell et al., 1966). This may be due to the high pressure in a gas reservoir. When applied to the near surface gas flow analysis, the same linearization procedure may give reasonable accuracy for gas flow in unsaturated zones due to small (a few percent) surface atmospheric pressure changes (Kidder, 1957). Nevertheless, the applicability of such a linearization approximation to a particular problem needs to be further studied.

The applicability of the linearized gas flow solutions to different situations depends mainly on how well an averaged formation pressure can be used to obtain a representative gas diffusivity term in (3.9) in the pressure disturbed zone. The conventional treatment, when Klinkenberg effects are ignored, is

$$\overline{P} \approx P_i \tag{4.1}$$

where P_i is the initial pressure of the system. This scheme may provide reasonable accuracy for certain pneumatic analysis (Shan, 1992) when the overall pressure changes are small with the system. However, using (4.1) to evaluate the diffusivity will introduce a large error when gas pressure changes are significant, such as in air sparging operations. A better scheme is to use a history-dependent, averaged pressure within the pressure changed domain, defined as:

$$\overline{P} \approx \frac{\sum A_j P_j}{\sum A_j} \tag{4.2}$$

where A_j is a controlled area at the geometric center of which the pressure was P_j at the previous time step, with the summation, $\sum A_j$, equal to the total area where pressure increases (or decreases) occur at the previous time. The idea is to use an averaged pressure which is close to the averaged pressure values to be solved for at the current time to evaluate the diffusivity instead of using an initial, constant pressure throughout. The procedure of evaluating a history-dependent diffusivity with (4.2) is similar to an explicit numerical solution scheme, but much more straightforward because all the P_j's are calculated analytically.

To demonstrate the new scheme for better estimation of the non-linear diffusivity term in the gas flow equations, we present the following comparison studies using a numerical model. A numerical code of multiphase flow, TOUGH2 (Pruess, 1991), is used here to examine the approximated transient gas flow solutions. The TOUGH2 code has been verified extensively for its accuracy in simulating gas flow in porous media (Pruess et al., 1996). The verification examples for gas flow with the Klinkenberg effect are provided in the next section. The testing problems concern transient gas flow in a linear semi-infinite and a radially infinite system. The systems are at single-phase gas flow under isothermal conditions. Constant gas mass injection rates are imposed at the inlet for the linear system and at the injection well for the radial system, respectively.

The parameters used for this comparison study are: porosity $\phi = 0.3$; permeability coefficient $k_\infty = 1\times10^{-15}$ m^2; Klinkenberg coefficient $b = 4.75\times10^4$ Pa; formation temperature $T = 25$ °C; compressibility factor $\beta = 1.18\times10^{-5}$ kg/Pa•m^3; gas viscosity $\mu_g = 1.84\times10^{-5}$ Pa•s; and initial pressure $P_i = 10^5$ Pa. The 1-D linear formation has a unit cross-section area, and the thickness of the radial system is 1 m. The inlet or well boundary conditions are: air mass injection rate $Q_m = 1\times10^{-6} - 1\times10^{-4}$ kg/s.

a. Linear Flow

Figure 4.1 presents the comparisons of the pressure profiles at 1 day calculated from the numerical (true) and analytical (approximate) solutions. At the lower injection rate of $q_{m,1} = 1\times10^{-6}$ kg/s.m^2 for the linear gas flow problem, the pressure increase in the system is relatively small at that time. Then the analytical solution using P_i for $\overline{P}$ gives excellent accuracy when compared with the numerical solution. However, as the injection rate increases ($q_{m,2} = 1\times10^{-5}$ kg/s.m^2), the gas pressure increases significantly. The analytical solution, with P_i as the averaged system pressure, gives poor accuracy, as shown in Figure 4.1. However under the same injection rate, the proposed scheme for evaluating the non-linear diffusivity using a history-dependent averaged pressure (4.2) results in excellent agreement with the numerical solution.

b. Radial Flow

The comparisons for the radial flow case are shown Figure 4.2 for the pressure profiles at 1 day from the numerical and analytical solutions, respectively. Similar to the linear flow case, at the lower injection rate of $Q_{m,1} = 1\times10^{-5}$ kg/s at the well, the pressure increase in the formation is relatively small after one day. The analytical solution using P_i for $\overline{P}$ gives good accuracy when compared with the numerical solution. As the injection rate increases ($Q_{m,2} = 1\times10^{-4}$ kg/s) the pressure increases by a factor of a few, as shown in Figure 4.2, the linearization with P_i as the averaged system pressure, introduces considerable errors to the predicted pressure profiles. However, the proposed scheme for evaluating the non-linear diffusivity using a history-dependent averaged pressure (4.2) again gives excellent accuracy as compared with the numerical solution.

5. Applications

In this section, several application examples will be given for the analytical solutions derived in Section 3. The application problems include: (1) checking the numerical scheme; (2) laboratory determination of permeability and Klinkenberg coefficient; (3) well test determination of permeability and Klinkenberg coefficient, and (4) laboratory test analysis.

5.1 Examination of Numerical Scheme

a. Steady State Flow

This is to examine the accuracy of the TOUGH2 formulation in simulating porous medium gas flow with the Klinkenberg effects. The problem concerns steady state gas flow across a linear rock column of 10 meters long. The system contains single-phase gas at isothermal condition, and a constant gas mass injection rate is imposed at the inlet of the column. The outlet end of the rock column is kept at a constant pressure. Eventually the system will reach steady state.

The formation and Klinkenberg properties were selected from a laboratory study of the welded tuff at Yucca Mountain (Reda, 1987). The parameters used are: porosity $\phi = 0.3$; permeability $k_\infty = 5\times10^{-19}$ m^2; Klinkenberg coefficient $b = 7.6\times10^5$ Pa; formation temperature $T = 25\ ^\circ$C; and compressibility factor $\beta = 1.18\times10^{-5}$ kg/Pa$\bullet$m^3; gas viscosity $\mu_g = 1.84\times10^{-5}$ Pa$\bullet$s. The boundary conditions are: air mass injection rate $Q_m = 1\times10^{-6}$ kg/s; and the outlet boundary pressure $P_L = 1\times10^5$ Pa.

A comparison of the pressure profile along the rock column from the TOUGH2 simulation and the exact, analytical solution (3.4) is shown in Figure 5.1, indicating that the TOUGH2 simulated pressure distribution is in excellent agreement with the analytical solution for this problem.

b. Transient Flow

This is to examine the capability of the TOUGH2 formulation in simulating transient gas flow with the Klinkenberg effects. The problem concerns gas injection into a well in a large horizontal, uniform, and isothermal formation. A constant gas mass injection rate is imposed at the well, and the initial formation pressure is constant.

The parameters used are: porosity ϕ = 0.3; permeability $k_\infty = 1\times10^{-15}$ m^2; Klinkenberg coefficient $b = 4.75\times10^4$ Pa; The air mass injection rate $Q_m = 1\times10^{-6}$ kg/s; the initial formation pressure $P_i = 1\times10^5$ Pa; the wellbore radius, r_w = 0.1 m; the formation thickness, h = 1 m; and all the other parameters are the same as in the steady state flow case above.

A comparison of the pressure profiles along the radial direction after ten days of injection from the TOUGH2 simulation and the analytical solution (3.27) is shown in Figure 5.2. Again, excellent agreement has been obtained for the transient flow problem.

5.2 Laboratory Determination of Permeability and Klinkenberg coefficient

The conventional method used in laboratory determination of the permeability and the Klinkenberg coefficient is using a plot of effective gas permeability k_g vs. inverse average pressure, $1/\bar{P}$ (instead of $1/P$), (Klinkenberg, 1941; Reda, 1987; and Persoff and Hulen, 1996). The use of $(P_0 + P_L)/2$ to represent P in Equation (1.1) appears questionable, especially since the pressure profile, even in 1-D flow, is not linear. Here we derive an alternative approach for determining both k_∞ and b from laboratory tests, based on the exact steady-state flow solutions discussed above.

By evaluating Equation (3.4) at x=0 one obtains, after some algebraic manipulation,

$$\frac{q_m \mu L}{\beta(P_0 - P_L)} = bk_\infty + k_\infty\left(\frac{P_0 + P_L}{2}\right) \tag{5.1}$$

To evaluate k_∞ and b experimentally, a series of measurements are made in which the outlet pressure P_L is held constant while P_0 is varied and q_m is measured. Then $Y = q_m \mu L / \beta(P_0 - P_L)$

is plotted against $X=(P_0+P_L)/2$. Then k_∞ and b are evaluated from the slope and intercept of the plot,

$$\frac{q_m \mu L}{\beta(P-P_L)} = bk_\infty + k_\infty\left(\frac{P+P_L}{2}\right) \tag{5.2}$$

For a radial sample, the corresponding equation is solved for two sets of measurements (P_1, Q_1), and (P_2, Q_2), P_e held constant:

$$b = -\frac{P_e + P_1}{2} + \frac{Q_1 \mu}{2\pi r_w h k_\infty \beta(P_1 - P_e)} ln\left(\frac{r_e}{r_w}\right) \tag{5.3}$$

where

$$k_\infty = \frac{\mu}{\pi r_w h \beta(P_2 - P_1)}\left\{\frac{Q_2}{P_2 - P_e} - \frac{Q_1}{P_1 - P_e}\right\} ln\left(\frac{r_e}{r_w}\right) \tag{5.4}$$

5.3 Well Test Determination of Permeability and Klinkenberg coefficient

The transient gas flow solutions of (3.19), (3.21), (3.27), and (3.29) can all be used to design well tests to determine both the gas permeability, k_∞, and the Klinkenberg coefficient, b. Here we give an example to demonstrate how to use the analytical solutions by a single well test of a constant mass rate pumping or injection with the line source solution, (3.27). The pumping or injection testing procedure is: (1) measure initial reservoir gas pressure, P_i ; (2) impose a constant mass pumping or injection rate at the well; and (3) measure several (at least two) wellbore pressures at different times (avoiding the early time after-flow or wellbore storage effects). The Klinkenberg coefficient, b, can be directly calculated as,

$$b = -\frac{P_n + P_i}{2} - \frac{\mu q_m}{4\pi h k_\infty \beta(P_n - P_i)} Ei\left(-\frac{r_w^2}{4\alpha t_n}\right) \tag{5.5}$$

and the permeability, k_∞, is determined from the following nonlinear algebraic equation,

$$P_j - P_n + \frac{\mu q_m}{2\pi h k_\infty \beta}\left\{\frac{Ei\left(-\frac{r_w^2}{4\alpha t_j}\right)}{(P_j - P_i)} - \frac{Ei\left(-\frac{r_w^2}{4\alpha t_n}\right)}{(P_n - P_i)}\right\} = 0 \tag{5.6}$$

In Equations (5.5) and (5.6), P_n and P_j are the wellbore pressures measured at two different times, $t = t_n$ and $t = t_j$, respectively.

This method can be demonstrated to analyze the simulated well test of the second example problem in Section 5.1 to determine the Klinkenberg coefficient, b, and gas permeability, k_∞. From the simulation, the well pressure $P_w = 1.05130\times10^5$ (Pa) at $t = 8.64\times10^4$ (sec.), and $P_w = 1.06976\times10^5$ (Pa) at $t = 8.64\times10^5$ sec. Substituting these pressure and time data into Equation (5.6), together with the parameters in Section 5.1, then the only unknown is k_∞ from the resulting non-linear equation. It can be easily solved using a bi-section method, which gives $k_\infty = 9.98\times10^{-16}$ m^2. Substituting this permeability value into (5.5) will give the Klinkenberg coefficient $b = 4.77\times10^5$ Pa. The actual values are $k_\infty = 1.0\times10^{-15}$ m^2 and $b = 4.75\times10^5$, and this indicates the proposed well test method is very accurate in determining these two Klinkenberg parameters.

5.4 Laboratory Test Analysis

a. Materials and Methods

Steady-state gas flow experiments were conducted to test the model and to evaluate k_∞ and b. Two rock core samples were obtained from well NEGU-17, in The Geysers geothermal field. Three cylindrical plugs, 15 mm in diameter were taken from the samples using a diamond core bit, and the cylinders ends were machined flat and parallel with lengths ranging from 9 to 11 mm.

The plugs were mounted into 2-inch long stainless steel tubing using Castall E-205 epoxy resin. They were then dried at 60° C for 5 days to remove all moisture. All three sample tubes were connected to a gas inlet manifold where nitrogen gas was applied at controlled pressures ranging from 120 kPa to 380 kPa.

Gas exiting from the sample flowed through a 1-meter-long length of horizontally mounted 3.175 mm o.d., 0.559 mm wall clear nylon tubing. To measure the gas flow rate, a slug of dyed water was injected into the tubing before it was connected to the sample tube, and the displacement of the slug was used to measure the gas flow rate. By monitoring the position of the slugs in the exit tubes, we were assured that steady state had been reached before measuring the flow rate.

Leaks that would normally be insignificant may be significant when measuring very low gas flows. An advantage of this experimental system is that any gas leak upstream of the sample would not cause any error, as long as the pressure is accurately measured. The gas flow which is to be monitored is at ambient pressure, so there is no driving force for it to leak and escape from the measurement tube. To test whether the technique of sealing the plugs into the stainless steel sample tube prevented gas from leaking past the sample, a dummy plug of aluminum was sealed into a stainless steel tube the same way and flow tested; no flow was observed.

b. Results and data analysis

The flow rate and pressure data are summarized in Table 2. These data will be interpreted according to the traditional Klinkenberg method and to the new model, referred as to exact Klinkenberg analysis, which is based on Equation (5.2). In both cases, the data of Table 2 are used to calculate derived quantities which are plotted as straight lines.

In the exact Klinkenberg analysis, the calculated quantities $X=(P_0+P_L)/2$ and $Y = q_m \mu L/\beta(P_0 - P_L)$ are summarized in Table 2 and are plotted in Figure 5.3 for the three samples.

In the traditional Klinkenberg analysis, the effective gas permeability k_g is plotted against the reciprocal of the arithmetic mean of the inlet and outlet pressures (Scheidegger, 1974). For a compressible gas, k_g is calculated by

$$k_g = \frac{2\mu L q_L^v P_L}{P_0^2 - P_L^2} \tag{5.7}$$

here the superscript v indicates that volumetric, not mass, flux, is to be evaluated at the outlet pressure.

Figure 5.4 plots the data calculated in Table 2, and Table 3 presents the calculated values of k_∞ and b derived from the linear plots, as well as the correlation coefficients. The values obtained by the two methods are close, although the traditional plot appears to have a better correlation.

By combining Equation (5.7) with (1.1), (2.3) and (2.4) and using $(P_o + P_L)/2$ to represent P in Equation (1.1), it is possible to show by algebraic manipulation that the values of k_∞ and b are the same whether the traditional method of Klinkenberg is used or Equation (5.1). In Table 3, the values of the constants differ slightly because there is experimental error in the determination of the straight lines. Analysis with two data points yields identical values.

When two constants are to be determined from more than two measurements (i.e., data are redundant), fitting the data to a linear equation using least squares generally provides the best estimates of the constants. But if more than one linearization is possible, the same data set will yield different results depending upon the linearization chosen (see, for example, Persoff and Thomas, 1988). It is tempting to prefer the linearization that yields the values of r^2 closer to unity. However, note that in the traditional method, the value of k_g (which is plotted as the dependent variable) calculated from (5.7) includes a factor of $1/(P_0 + P_L)$ which is just double the independent variable. This artificially increases the value of r^2.

The method of analysis developed in this paper therefore confirms the acceptability of the traditional method of calculating the constants from 1-D steady flow tests. Additionally, it permits the values to be calculated from well tests and tests in geometries other than 1-D linear flow. It also permits the calculation of pressure profiles for a variety of flow geometries.

Table 2. Steady-state gas flow measurements on plugs of The Geysers greywacke from well NEGU-17.

	raw data			quantities calculated for traditional analysis		quantities calculated for Exact analysis	
sample dimensions	inlet pressure (Pa)	outlet pressure (Pa)	volume flow rate at exit pressure (m^3)	inverse average pressure (Pa^{-1})	k_g (m^2)	$X = \frac{P_0 + P_L}{2}$ (Pa)	$\frac{q_m \mu L}{\beta(P_0 - P_L)}$ (N)
sample 36 A =1.79E-04 m^2 L = 9.07E-03 m	2.18E+05	9.88E+04	4.48E-11	6.31E-06	2.07E-19	1.59E+05	3.28E-14
	2.64E+05	9.88E+04	6.24E-11	5.51E-06	1.82E-19	1.82E+05	3.30E-14
	3.05E+05	9.88E+04	7.88E-11	4.95E-06	1.66E-19	2.02E+05	3.34E-14
	3.40E+05	9.87E+04	9.26E-11	4.56E-06	1.53E-19	2.19E+05	3.35E-14
	3.80E+05	9.93E+04	1.09E-10	4.17E-06	1.42E-19	2.40E+05	3.40E-14
	1.42E+05	9.95E+04	1.54E-11	8.29E-06	2.65E-19	1.21E+05	3.20E-14
sample 9a A =1.83E-04 m^2 L =1.04E-02 m	2.18E+05	9.88E+04	5.41E-11	6.31E-06	2.79E-19	1.59E+05	4.43E-14
	2.64E+05	9.88E+04	7.58E-11	5.51E-06	2.47E-19	1.82E+05	4.48E-14
	3.05E+05	9.88E+04	9.53E-11	4.95E-06	2.24E-19	2.02E+05	4.53E-14
	3.40E+05	9.87E+04	1.11E-10	4.56E-06	2.05E-19	2.19E+05	4.50E-14
	3.80E+05	9.93E+04	1.31E-10	4.17E-06	1.91E-19	2.40E+05	4.58E-14
	1.42E+05	9.95E+04	1.81E-11	8.29E-06	3.49E-19	1.21E+05	4.22E-14
sample 9b A =1.83E-04 m^2 L =1.04E-02 m	2.18E+05	9.88E+04	6.16E-11	6.31E-06	3.18E-19	1.59E+05	5.04E-14
	2.64E+05	9.88E+04	8.66E-11	5.51E-06	2.82E-19	1.82E+05	5.12E-14
	3.05E+05	9.88E+04	1.10E-10	4.95E-06	2.58E-19	2.02E+05	5.20E-14
	3.40E+05	9.87E+04	1.25E-10	4.56E-06	2.31E-19	2.19E+05	5.07E-14
	3.80E+05	9.93E+04	1.52E-10	4.17E-06	2.22E-19	2.40E+05	5.32E-14
	1.42E+05	9.95E+04	2.07E-11	8.29E-06	3.99E-19	1.21E+05	4.82E-14
	1.20E+05	9.91E+04	1.01E-11	9.12E-06	4.30E-19	1.10E+05	4.71E-14

Table 3. Analysis results of the laboratory tests.

	traditional			exact		
sample	$k_\infty(m^2)$	b(Pa)	r^2	$k_\infty(m^2)$	b(Pa)	r^2
36	1.66E-20	1.81E+06	1.00E+00	1.61E-20	1.88E+06	9.88E-01
9a	3.15E-20	1.23E+06	9.99E-01	2.75E-20	1.43E+06	9.33E-01
9b	4.38E-20	9.75E+05	9.99E-01	4.02E-20	1.08E+06	9.28E-01

where r^2 is correlation coefficient.

6. Concluding remarks

A general gas flow governing equation including the Klinkenberg effect has been derived by introducing a new pressure variable. Based on this new flow governing equation, a set of new analytical solutions have been developed for analyzing steady state and transient gas flow through porous media with Klinkenberg effects. As an extension of this work, additional analytical solutions for 1-D, 2-D and 3-D gas flow with the Klinkenberg effect can be readily derived. These analytical solutions will find their applications in analyzing gas flow and determining soil flow properties in the unsaturated zone or in laboratory tests, where the Klinkenberg effects cannot be ignored.

In an effort to determine the condition under which the linearized gas flow equation may be applicable in applications, a numerical method is used to examine the predictions from the approximate analytical solutions. It has been found that the conventional linearization procedure, using an averaged gas pressure for the diffusivity term, will result in acceptable solutions when the overall pressure variations in the system are small. However, the linearization assumption may introduce considerable errors when pressure changes are significantly different from the ambient condition. In this case, we propose a new evaluation procedure for the diffusivity term using analytical solutions, which will still give accurate solutions under high pressure disturbed conditions.

In order to demonstrate their applications, the new analytical solutions have been used to verify the numerical solutions of gas flow, which include the Klinkenberg effect. Several new laboratory and field testing techniques are derived based on the analytical solutions for determining the Klinkenberg parameters of porous medium gas flow. These new laboratory and field test analysis methods are very easy to implement and more accurate to use. One of the proposed laboratory methods has been applied to laboratory testing results in determining absolute permeability and Klinkenberg constants. The transient test analysis method is illustrated using a simulated well test result.

Notation

Roman Letters

A	cross-section area, m^2.
b	Klinkenberg coefficient, Pa.
$\vec{g}$	gravity vector, m/s^2.
h	formation thickness, m.
k_g	effective gas permeability, m^2.
k_∞	absolute permeability, m^2.
L	length of linear flow systems, m.
M_g	molecular weight of gas.
P	gas pressure, Pa.
P_0, P_1, P_2	gas pressure at inlet boundaries of linear flow systems, Pa.
P_b	gas pressure function (2.6), Pa.
P_e	gas pressure at outer boundaries of radial flow systems, Pa.
P_i	initial gas pressure, Pa.
P_L	gas pressure at outlet boundaries of linear flow systems, Pa.
P_w	wellbore gas pressure, Pa.
$\overline{P}$	averaged (constant) gas pressure, Pa.
$\overline{P_b}$	averaged gas pressure function ($= \overline{P} + b$), Pa.
q_m, q_{m0}, q_1, q_2	gas mass injection or pumping flux, $kg/(s.m^2)$.
q_L^v	volumetric gas injection flux, $m^3/(s.m^2)$, measured at outlet pressure.
Q_m, Q_1, Q_2	gas mass injection or pumping rate, kg/s.
r	radial distance, m.
R	universe gas constant.
r_e	radial distance of outer boundaries of radial flow systems, m.

r_w	wellbore radius, m.
T	temperature, °C.
t	time, s.
$\vec{v}$	Darcy's velocity of gas phase, m/s.

Greek Letters

α	gas diffusivity (3.9).
β	compressibility factor (2.4).
ϕ	porosity.
μ	viscosity, Pa•s.

Subscripts

g	gas.
i	initial.
L	outlet at $x=L$.
m	mass
0	inlet at x=0.
w	well.

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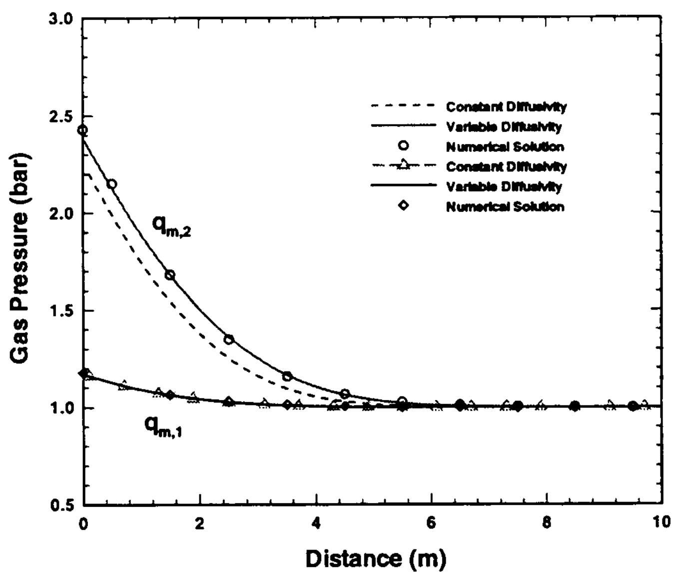

Figure 4.1 Comparison of gas pressure profiles in a linear semi-infinite system at 1 day, calculated using the numerical and the analytical solutions.

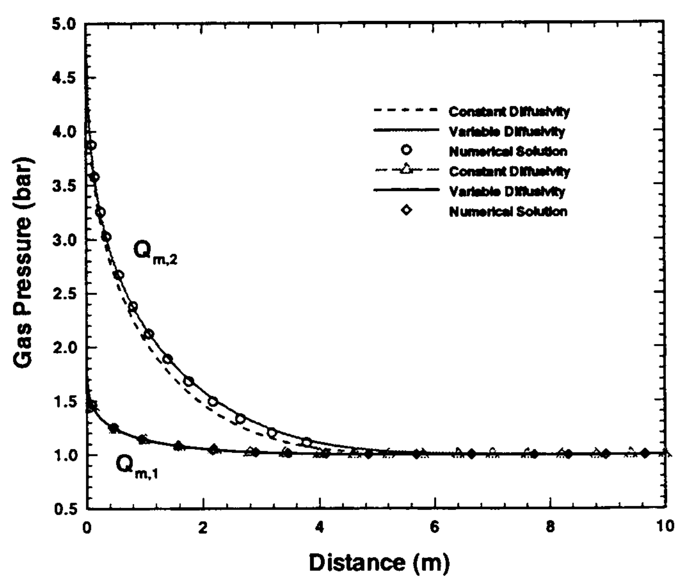

Figure 4.2 Comparison of gas pressure profiles in a radially infinite system at 1 day, calculated using the numerical and the analytical solutions.

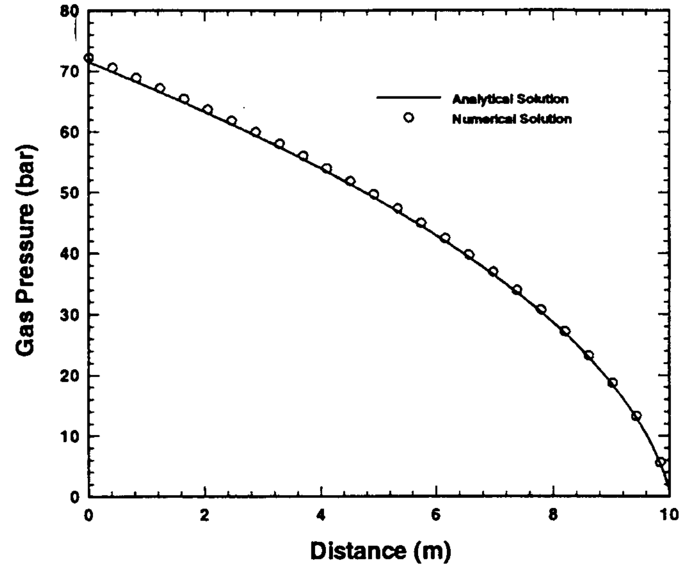

Figure 5.1 Comparison of the analytical and the numerical solutions for steady-state gas flow in a finite linear system.

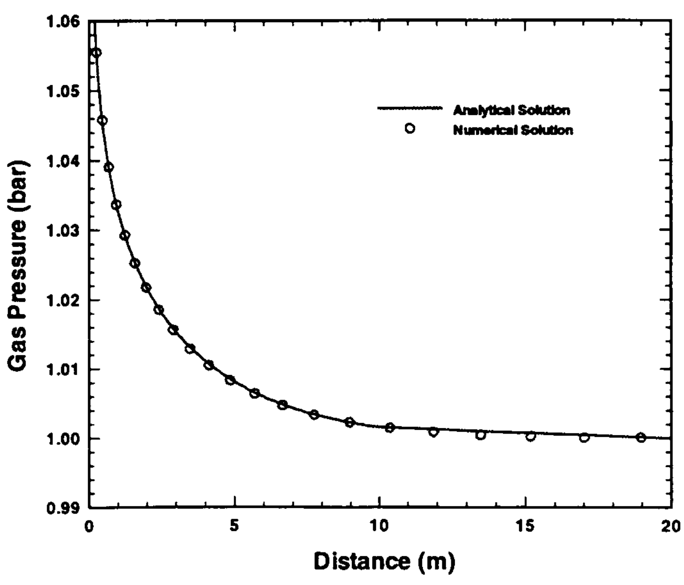

Figure 5.2 Comparison of the analytical and the numerical solutions for transient gas flow in a radial system.

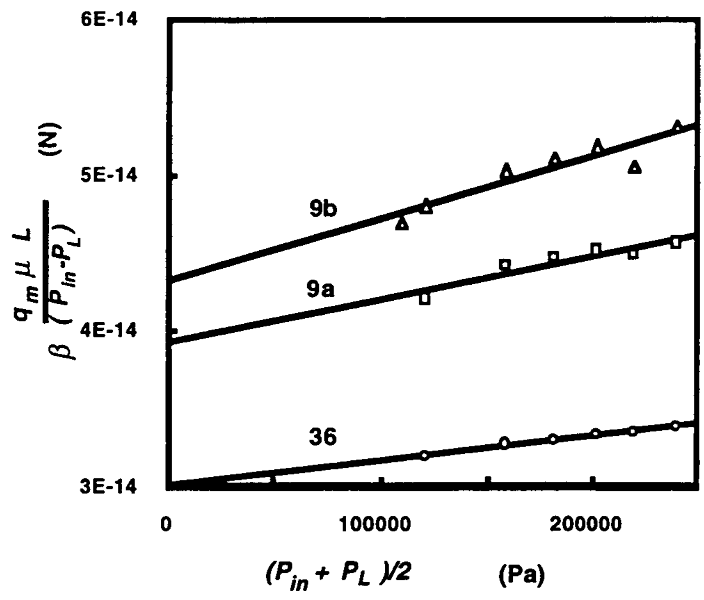

Figure 5.3 Exact Klinkenberg analysis plot for the three test samples.

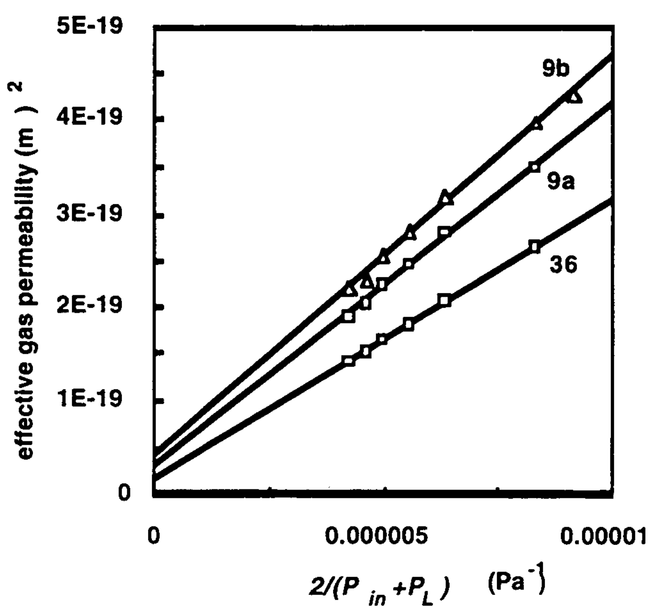

Figure 5.4 Traditional Klinkenberg analysis plot for the three test samples.